AS-level Maths, Sample Paper

Set Yπ – Fully Worked Solutions

Metric Academy

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Solutions

Question 1 Solution

We are given:

- Line l_1: $3x + 2y - 5 = 0$
- Line l_2: $y = mx - 4$
- $l_1 \perp l_2$ (they are perpendicular)

(a) Find the value of m

Rearrange the equation of l_1 to the form $y = mx + c$:

$$\begin{aligned} 3x + 2y - 5 &= 0 \\ 2y &= -3x + 5 \\ y &= -\frac{3}{2}x + \frac{5}{2} \end{aligned}$$

So the gradient of l_1 is $-\frac{3}{2}$.

Since the lines are perpendicular, the product of their gradients is -1:

$$\left(-\frac{3}{2}\right) \cdot m = -1 \quad \Rightarrow \quad m = \frac{2}{3}$$

Answer: $m = \frac{2}{3}$

(b) Find the x-coordinate of point P

Now substitute $m = \frac{2}{3}$ into the equation of l_2:

$$y = \frac{2}{3}x - 4$$

To find the point of intersection with l_1, solve the system:

$$\begin{aligned} 3x + 2y &= 5 \quad \text{(rearranged from } l_1\text{)} \\ y &= \frac{2}{3}x - 4 \end{aligned}$$

Substitute the second equation into the first:

$$\begin{aligned}3x + 2\left(\frac{2}{3}x - 4\right) &= 5\\ 3x + \frac{4}{3}x - 8 &= 5\\ \left(3x + \frac{4}{3}x\right) &= 13\\ \left(\frac{9}{3}x + \frac{4}{3}x\right) &= 13\\ \frac{13}{3}x &= 13\\ x &= 3\end{aligned}$$

Answer: $x = 3$

Question 2 Solutions

(i) Solve $9x^2 = 3\sqrt{x}$

Rewriting the equation:

$$9x^2 - 3\sqrt{x} = 0$$

Let $u = \sqrt{x}$, so $x = u^2$. Then:

$$9(u^2)^2 - 3u = 0 \Rightarrow 9u^4 - 3u = 0$$

Factor:

$$3u(3u^3 - 1) = 0$$

Solving each factor:

$$3u = 0 \Rightarrow u = 0 \Rightarrow x = u^2 = 0$$

$$3u^3 - 1 = 0 \Rightarrow u^3 = \frac{1}{3} \Rightarrow u = \sqrt[3]{\frac{1}{3}} \Rightarrow x = u^2 = \sqrt[3]{\frac{1}{9}}$$

Final Answers: $x = 0$ and $x = \sqrt[3]{\frac{1}{9}}$

(ii) Solve $c^4 - 5c^2 - 24 = 0$

Let $u = c^2$. Then:

$$u^2 - 5u - 24 = 0$$

Use the quadratic formula:

$$u = \frac{5 \pm \sqrt{(-5)^2 + 4 \cdot 1 \cdot 24}}{2 \cdot 1} = \frac{5 \pm \sqrt{121}}{2} = \frac{5 \pm 11}{2}$$

$$u = 8 \quad \text{or} \quad u = -3$$

So:

$$c^2 = 8 \Rightarrow c = \pm 2\sqrt{2} c^2 = -3 \quad \text{(no real solution)}$$

Final Answers: $c = \pm 2\sqrt{2}$

Question 3 Solutions

We are given:

$$\vec{XY} = 4\vec{i} + 2\vec{j}, \quad \vec{XZ} = 10\vec{i} - 8\vec{j}$$

(a) Find $\vec{YZ}$

$$\vec{YZ} = \vec{XZ} - \vec{XY} = (10\vec{i} - 8\vec{j}) - (4\vec{i} + 2\vec{j}) = 6\vec{i} - 10\vec{j}$$

Answer: $\vec{YZ} = 6\vec{i} - 10\vec{j}$

(b) Find $|\vec{YZ}|$ as a simplified surd

$$|\vec{YZ}| = \sqrt{6^2 + (-10)^2} = \sqrt{36 + 100} = \sqrt{136} = \sqrt{4 \cdot 34} = 2\sqrt{34}$$

Answer: $|\vec{YZ}| = 2\sqrt{34}$

(c) Find $\vec{XW}$, where $YW : WZ = 2 : 3$

$$\vec{YW} = \frac{2}{5} \cdot \vec{YZ} = \frac{2}{5}(6\vec{i} - 10\vec{j}) = \frac{12}{5}\vec{i} - 4\vec{j}$$

$$\vec{XW} = \vec{XY} + \vec{YW} = (4\vec{i} + 2\vec{j}) + \left(\frac{12}{5}\vec{i} - 4\vec{j}\right) = \frac{32}{5}\vec{i} - 2\vec{j}$$

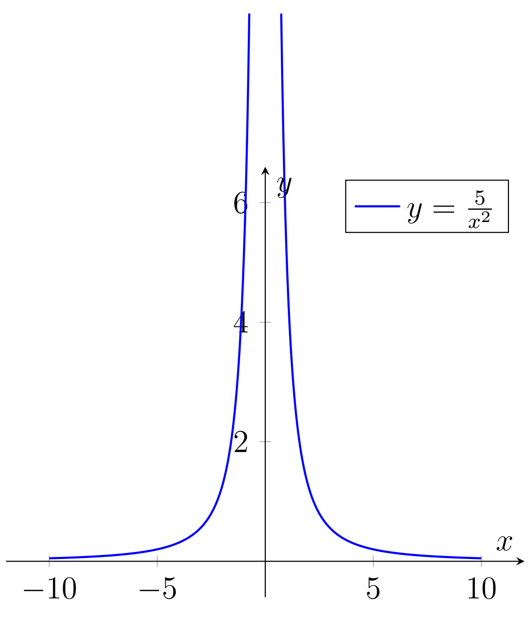
Answer: $\vec{XW} = \frac{32}{5}\vec{i} - 2\vec{j}$

Question 4 Solutions

(a) Sketch of the curve $y = \frac{m}{x^2}$

For a positive constant m, the graph of $y = \frac{m}{x^2}$ has the following properties:

- Defined for all $x \neq 0$
- $y > 0$ for all $x \neq 0$
- Vertical asymptote at $x = 0$
- Horizontal asymptote at $y = 0$
- Symmetric about the y-axis



(b) Solve the inequality $\frac{25}{x^2} < 1$

Multiply both sides by x^2 (valid since $x^2 > 0$, and $x \neq 0$):

$$25 < x^2 \quad \Rightarrow \quad x^2 > 25 \Rightarrow x > 5 \quad \text{or} \quad x < -5$$

Final Answer: $x \in (-\infty, -5) \cup (5, \infty)$

In standard set notation:

$$\{x \in \mathbf{R} : x < 5,\ \text{or } x > 5\}.$$

Question 10 Solutions

(a) Show that $x^2 - 3x - 2 = 0$

Given:

$$\log_2(x+1) + \log_2(x-2) = 1 + 2\log_2(x)$$

Use log laws:

LHS:

$$\log_2[(x+1)(x-2)] = \log_2(x^2 - x - 2)$$

RHS:

$$1 + 2\log_2(x) = \log_2(2) + \log_2(x^2) = \log_2(2x^2)$$

So:

$$\log_2(x^2 - x - 2) = \log_2(2x^2) \Rightarrow x^2 - x - 2 = 2x^2 \Rightarrow -x^2 - x - 2 = 0 \Rightarrow x^2 - 3x - 2 = 0$$

Proven.

(b)(i) Solve $x^2 - 3x - 2 = 0$

Use the quadratic formula:

$$x = \frac{3 \pm \sqrt{9+8}}{2} = \frac{3 \pm \sqrt{17}}{2}$$

Solutions:

$$x = \frac{3+\sqrt{17}}{2}, \quad x = \frac{3-\sqrt{17}}{2}$$

(b)(ii) Which root is not a solution to the original equation?

We require:

$$x > 0, \quad x+1 > 0, \quad x-2 > 0 \Rightarrow x > 2$$

Only:

$$x = \frac{3+\sqrt{17}}{2} \approx 3.561$$

satisfies this domain.

Answer: $x = \frac{3-\sqrt{17}}{2}$ is not a solution because it does not lie in the domain of the logarithmic functions.